Political Statistics

A normal distribution has Mu = 40 and standard deviation = 8

1. Describe the distribution of sample means based on samples of n = 16 selected from this population

2. Of all the possible samples of n = 16, what proportion will have sample means greater than 42?

3. Of all the possible samples of n = 16, what proportion will have sample means less than 39?

Answer

Given that X~N(40,8)

1. The distribution of $\bar{X}$ is N(μ , $s\ /\sqrt{n}$)

   Thus $\bar{X} \sim N(40, 2)$

2. Standardizing the variable using $Z = \dfrac{\bar{X} - m}{s\ /\sqrt{n}}$ and from normal tables

   $$P\left(\bar{X} \geq 42\right) = P\left(\frac{\bar{X} - 40}{2} \geq \frac{42 - 40}{2}\right) = P(Z > 1) = 0.1587$$

   Thus 15.87% of samples will have sample means greater than 42?

3. Of all the possible samples of n = 16, what proportion will have sample means less than 39?

   $$P\left(\bar{X} \leq 39\right) = P\left(\frac{\bar{X} - 40}{2} \leq \frac{39 - 40}{2}\right) = P(Z < -0.5) = 0.3085$$

Thus 30.85% of samples will have sample means less than 39?

B. A statistics instructor would like to know whether it is worthwhile to require students to do weekly homework assignments. For one section of the statistics course, homework is assigned, collected and graded each week. For another section, the same problems are suggested each week, but the students are not required to turn in their homework. At the end of the semester, all students take the same final exam. The grade distribution for the two sections are as follows:

	A	B	C	D	E
Homework	12	15	17	5	1
No Homework	12	21	28	25	14

Do these data indicate a significant difference between the grade distributions for students with homework verses students with no homework?
Test with alpha = .05

Answer

The null hypothesis under consideration

H0: There is no significant difference in the mean grade of two groups of students
H1: There is significant difference in the mean grade of two groups of students

Test Statistic used is students t

$$t = \frac{\bar{X}_1 - \bar{X}_2}{S}\sqrt{\frac{n_1 n_2}{n_1 + n_2}} \sim t_{n_1+n_2-2} \text{ where } S = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}$$

Rejection criteria: reject the null hypothesis, if the calculated value of t is grater than the critical value of t with 8 d.f at 0.05 significance level
Details

t-Test: Two-Sample Assuming Equal Variances		
	Home Work	*No home work*
Mean	10	20
Variance	46	47.5
Observations	5	5
Pooled Variance	46.75	
Hypothesized Mean Difference	0	
df	8	
t Stat	-2.31248645	
P(T<=t) one-tail	0.024748295	
t Critical one-tail	1.859548033	
P(T<=t) two-tail	0.049496591	
t Critical two-tail	2.306004133	

Conclusion

Since calculated value of t is greater than the critical value, we reject the null hypothesis